

Corrigendum

Corrigendum to “Tail behaviour of the busy period of a GI/GI/1 queue with subexponential service times” [Stochastic Process. Appl. 111 (2004) 237–258]

Daryl J. Daley^a, Claudia Klüppelberg^{b,*}, Yang Yang^c

^a *Department of Mathematics and Statistics, The University of Melbourne, Parkville, VIC 3052, Australia*

^b *Centre for Mathematical Sciences, and Institute for Advanced Study, Technische Universität München, 85748 Garching, Germany*

^c *School of Mathematics and Statistics, Nanjing Audit University, Nanjing City, Jiangsu Province, 210029, PR China*

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Abstract

The purpose of this note is to correct an error in Baltrunas et al. (2004) [1], and to give a more detailed argument to a formula whose validity has been questioned over the years. These details close a gap in the proof of Theorem 4.1 as originally stated, the validity of which is hereby strengthened.

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All details refer to [1]. On p. 250, line-3, the moment generating function of the truncated random variable $V = UI_{\{U < y\}}$ is given, but a term was omitted in error. The line should read, correctly, for given $y > -\mu$

$$\begin{aligned}\tilde{f}(s) &= \int_{-\infty}^y e^{sv} dP(V \leq v) + P(U > y) \\ &= E[e^{sU} | U \leq y]P(U \leq y) + P(U > y), \quad s \in \mathbb{R}.\end{aligned}$$

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* Corresponding author.

E-mail addresses: dndaley@gmail.com (D.J. Daley), cklu@ma.tum.de (C. Klüppelberg), yyangmath@gmail.com (Y. Yang).

URL: <http://www-m4.ma.tum.de> (C. Klüppelberg).

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We follow the missing term $P(U > y)$ through the argument.

After Eq. (4.11) we decompose $\tilde{f}(s)$ for $sy > 1$ into

$$\tilde{f}(s) = J_0 + J_1 + P(U > y).$$

In the following we find $J_0 = 1 + O(1)s^2$ and $J_1 = O(1)s^2$. From Lemma 3.6(b) we know that the moment index $\kappa \geq \beta$, which in turn is greater than 2 by Condition B(ii). Consequently, $E[X^2] < \infty$. Since $U = X - Y - \mu$, this implies that U has finite second moment. Invoking Markov's inequality, we obtain that $P(U > y) \leq P(U > 1/s) \leq s^2 E[U^2] = O(1)s^2$. Substituting this into (4.11) yields (4.14) as in the paper.

The second point of concern is the last line of p. 252. We give here a more detailed calculation showing that it holds for given y for sufficiently small $s > 0$. Since the i.i.d. random variables V_k^s have finite variance, Chebychev's inequality gives

$$\begin{aligned} P\left(\sum_{k=1}^n V_k^s > t\right) &= P\left(\sum_{k=1}^n V_k^s - nE[V^s] > t - nE[V^s]\right) \\ &\leq \frac{n\text{Var}(V^s)}{(t - nE[V^s])^2} \leq \frac{nE[(V^s)^2]}{(t - nE[V^s])^2}, \end{aligned}$$

and the right-hand side above is bounded by $(n/t^2)E[(V^s)^2]$, as asserted on the last line of p. 252, provided that $E[V^s] < 0$. To show that for large enough y we have $E[V^s] < 0$ for all sufficiently small $s > 0$, observe that, no matter what finite $y > 0$ is given, we have $E[V^s]_{|s=0+} = E[UI_{\{U < y\}}] < 0$, and $E[V^s]$ has a finite derivative in s for all small enough $s > 0$. This implies that $E[V^s] < 0$ for all sufficiently small $s > 0$, as required for our argument. This has shown in fact that the probability above is bounded by $(n/t^2)\text{Var}(V^s)$, which is tighter than what we claimed originally.

References

- [1] A. Baltrunas, D.J. Daley, C. Klüppelberg, Tail behaviour of the busy period of GI/G/1 queue with subexponential service times, *Stochastic Process. Appl.* 111 (2) (2004) 237–258.